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Hartl, R.F.; Kort, P.M.

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Optimal input substitution of a firm facing an environmental constraint [1,2]

Richard F. Hartl [a,*], Peter M. Kort [b]

[a] *Institute of Management, University of Vienna, Brünnerstraße 72, A-1210 Vienna, Austria*
[b] *Department of Econometrics and CentER, Tilburg University, P.O. Box 90 153, NL-5000 LE Tilburg, The Netherlands*

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Abstract

In this paper we consider the dynamic behavior of a firm subject to environmental regulation. As a social planner the government wants to reduce the level of pollution. To reach that aim it can, among others, set an upper limit on polluting emissions of the firm. The paper determines how this policy instrument influences the firm's decisions concerning investments, abatement efforts, and the choice whether to leave some capacity unused or not. The abatement process is modeled as input substitution rather than end-of-pipe. Using standard control theory in determining the firm's optimal dynamic investment decisions it turns out that it is always optimal to approach a long run optimal level of capital. In some cases, this equilibrium is reached within finite time, but usually it will be approached asymptotically. Different scenarios are considered, ranging from attractive clean input to unattractive clean input, and from a mild emission limit to a very tight one. It is shown that for large capital stocks and/or when marginal cash flow per unit of emissions is larger for the dirty input than for the clean input, it can be optimal to actually leave some production capacity unused. Also, since the convex installation costs suggest to spread investments over time, it can happen that investment in productive capital is positive although capacity remains unused. © 1997 Elsevier Science B.V.

Keywords: Optimal control; Environment; Dynamic programming; Production and investment; Input substitution

1. Introduction

Currently, an important problem of a firm is how to react on environmental regulation. Acting as a social planner the government wants to reduce emissions generated by the firm's production process. To reach this aim it can use instruments like imposing an upper limit on pollution generated by the firm, imposing a pollution tax, or it can create a market where the firm must buy permits in order to be allowed to pollute the environment.

According to Jorgenson and Wilcoxen (1990) possible firm's responses to environmental regulation are investing in abatement technology, i.e. the use of special devices to treat wastes after they have been generated, and changing the production process to reduce emissions. The first response is commonly

* Corresponding author.

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known as end-of-pipe abatement and is often the choice when existing firms have to meet newly imposed standards.

Compared to end-of-pipe abatement a more modern approach is changing the production process in order to reduce emissions. The least disruptive way of doing this is to switch to cleaner inputs. A straightforward example is available from electric utilities, namely switching from high-sulphur to low-sulphur coal in order to comply with restrictions on sulphur dioxide emissions (Jorgenson and Wilcoxen, 1990).

In this paper we want to establish optimal dynamic firm behavior, while the firm's emissions are limited. It is assumed that in the short run the firm has two possibilities to reduce its own emissions: it can decrease production by leaving some production capacity unused, or it can use a clean but expensive input instead of a dirty and cheap input in the production process. In the long run the firm can reckon with this emission limit by adjusting its investment program.

Dynamic firm behavior under an emission limit was also subject of study in Hartl and Kort (1996), Kort (1994) and Xepapadeas (1992). Xepapadeas analyzed end-of-pipe abatement rather than input substitution, while contrary to this work, his analysis is limited to solutions where the environmental constraint is always binding. Here we consider transitions from situations where the constraint is binding to situations where the constraint is non-binding and the other way round. Under which scenarios do these transitions occur and how are these affected by the level of the emission limit? In the present paper, input substitution is modeled more precisely than in Hartl and Kort (1996) and Kort (1994) in the sense that here we exactly specify the amount of both inputs being used at any time, rather than identifying a variable for abatement expenditures which represents money lost due to input substitution. Moreover, in this contribution the above-mentioned papers are extended by allowing the firm to leave some capacity unused in the production process.

The model of the firm is specified as an optimal control problem (Section 2), which is solved by performing a two step approach. The two steps are identified in Section 3, while in Section 4 Step 1 is solved by determining the optimal use of the inputs and the production capacity for every given level of the stock of capital goods. The optimal investment program is established in Section 5, given the optimal use of inputs and production capacity. Section 6 concludes the paper.

2. The model

Consider a firm having a stock of capital goods, K, through which it can produce output, $Q(K)$. It is reasonable to assume that

$$Q(0)=0, \qquad Q'(K)>0, \qquad Q''(K)<0. \tag{1}$$

Furthermore, we assume that $Q(K)$ is bounded, i.e., $Q'(\infty)<\infty$.

The actual production level, q, must therefore satisfy

$$0 \le q \le Q(K). \tag{2}$$

Production of this q requires the use of a variable input, where the firm has the possibility to use a *cheap and dirty* input, F, and/or an *expensive and clean* input, A. As practical examples we think of the possibility of choice between cheap and dirty high sulphur coal and expensive and clean low sulphur coal by electric utilities or if in some process harmful exhaust fumes are generated (e.g. dioxins when burning some material e.g. waste), one can usually reduce these emissions by increasing the temperature in this process by using more or different oil. In order to focus on the effects, this kind of substitutions, i.e. between F and A, have on the firm's investment behavior, substitution possibilities between the (short term) fixed input K and the variable inputs F and A are ruled out.

Let us assume the following relationship between output q and total input $F+A$:

$$q=G(F+A), \tag{3}$$

where G is strictly concave, i.e.,

$$G'>0, \qquad G''<0, \qquad G(0)=0. \tag{4}$$

In case this firm produces with full capacity, we obtain from Eq. (2) that $q=Q(K)$. In combination with Eq. (3) this yields the following relation between $F+A$ and K:

$$F+A=S(K), \tag{5}$$

where $S(K)$ is defined as

$$S(K) = G^{-1}(Q(K)). \tag{6}$$

We assume that $S(K)$ is convex, with $S'(\infty) < \infty$, which means that the total variable input increases more than proportionally with the short term fixed input K as long as the firm produces with full capacity.

It is assumed that the production process generates emissions E, which depend on the inputs F and A in a linear way:

$$E = b_1 F + b_2 A, \tag{7}$$

where $b_1 > b_2$ since A is the 'clean' input.

Let the unit price of the two inputs F and A be given by v_1 and v_2, respectively with $v_1 < v_2$ since the clean input A is more expensive. Therefore, $A = 0$ is associated with the input choice of a profit maximizing firm in absence of any form of environmental regulation.

Of course, both inputs are always non-negative:

$$F \geq 0, \qquad A \geq 0. \tag{8}$$

The government reduces pollution by imposing a fixed upper bound on emissions. Let Z be the maximum permitted volume at time t. The emission constraint for the firm at each instant of time is

$$E(A, F) \leq Z. \tag{9}$$

Because capacity can remain unused, this constraint can always be met by a sufficiently low value of q, no matter how large K and therefore $Q(K)$ is.

The capital stock can be increased by productive investments I where investment costs equal $C(I)$ for which we assume

$$C(0) = 0, \qquad C'(I) > 0, \qquad C''(I) > 0. \tag{10}$$

On the other hand, the capital stock decreases by depreciation at rate a.

We assume that the firm is 'small' in the sense that its product can be sold on the market at a fixed market price p.

If we further assume that the firm maximizes cash flow and discounts against rate r ($r > 0$ and constant), we obtain the following dynamic model of the firm:

$$\max_{I, q, A, F} \int_0^\infty e^{-rt}\{pq - v_1 F - v_2 A - C(I)\}\,dt \tag{11a}$$

s.t.

$$\dot{K} = I - aK, \qquad K(0) = K_0, \tag{11b}$$

$$0 \leq q \leq Q(K), \tag{11c}$$

$$q = G(F + A), \tag{11d}$$

$$F \geq 0, \qquad A \geq 0, \tag{11e}$$

$$b_1 F + b_2 A \leq Z. \tag{11f}$$

We have two additional assumptions. First, we impose that it always pays to use full capacity if the emission constraint is absent, i.e.,

$$G'(\infty) > v_1/p. \tag{12}$$

Notice that the firm only uses the cheap input F, in case there is no environmental regulation.

Second, we assume that the marginal revenue is sufficiently large for $K = 0$ so that it pays to start producing under full capacity using the cheap input:

$$pQ'(0) > (r + a)C'(0) + v_1 S'(0). \tag{13}$$

3. A two step approach

It is convenient to define $\bar{K} = \bar{K}(Z)$ as the largest capital stock for which producing with using only the dirty input F, i.e. $A = 0$, is in accordance with the emission limit, when the firm uses all of its production capacity, i.e., when $q = Q(K)$:

$$A = 0, \qquad F = \bar{F} = \frac{Z}{b_1}, \qquad G\left(\frac{Z}{b_1}\right) = Q(\bar{K}). \tag{14}$$

Furthermore, we define $\tilde{K}$ as the largest K for which full capacity can be used while the emission constraint is still satisfied:

$$F = 0, \qquad A = \tilde{A} = \frac{Z}{b_2}, \qquad G\left(\frac{Z}{b_2}\right) = Q(\tilde{K}). \tag{15}$$

Clearly, $\tilde{K} > \bar{K}$ and $\tilde{A} > \bar{F}$ since $b_2 < b_1$.

Since the control variables q, F, and A do not enter the system dynamics, this model can be treated by applying a two step approach (cf. Hartl, 1988):

Step 1: For every fixed K, solve the static problem of maximizing revenue minus expenses on the variable inputs F and A subject to the restrictions below:

$$\{pG(F+A) - v_1 F - v_2 A\} \rightarrow \max_{F,A} \tag{16a}$$

s.t.

$$b_1 F + b_2 A \leq Z, \tag{16b}$$

$$A \geq 0, \qquad F \geq 0, \tag{16c}$$

$$G(F+A) \leq Q(K). \tag{16d}$$

Step 2: With $q(K)$, $A(K)$ and $F(K)$ computed in Step 1, solve the following control problem:

$$\max_I \int_0^\infty e^{-rt}\{pq(K) - v_1 F(K) - v_2 A(K) - C(I)\}\,dt \tag{17a}$$

s.t.

$$\dot{K} = I - aK, \qquad K(0) = K_0. \tag{17b}$$

4. Solution of Step 1

The solution of the static Step 1 problem can be obtained using the Kuhn–Tucker conditions

$$L = \{pG(F+A) - v_1 F - v_2 A\} + \mu_1[Q(K) - G(F+A)] + \mu_2[Z - b_1 F - b_2 A] + \mu_3 A + \mu_4 F,$$

$$L_A = \{pG'(F+A) - v_2\} - \mu_1[G'(F+A)] - \mu_2[b_2] + \mu_3 = 0, \tag{18a}$$

$$L_F = \{pG'(F+A) - v_1\} - \mu_1[G'(F+A)] - \mu_2[b_1] + \mu_4 = 0, \tag{18b}$$

$$\mu_1[Q(K) - G(F+A)] = 0, \tag{18c}$$

$$\mu_2[Z - b_1 F - b_2 A] = 0, \tag{18d}$$

$$\mu_3 A = \mu_4 F = 0, \tag{18e}$$

$$\mu_1 \geq 0, \quad \mu_2 \geq 0, \quad \mu_3 \geq 0, \quad \mu_4 \geq 0. \tag{18f}$$

Combining Eq. (18a) and Eq. (18b),

$$L_A - L_F = \{v_1 - v_2\} + \mu_2[b_1 - b_2] + \mu_3 - \mu_4 = 0, \tag{19}$$

we obtain the following lemma:

Lemma 1. *It cannot be optimal that $\mu_2 = \mu_3 = 0$ at the same time, i.e. that the clean input is used, $A > 0$, and the emission limit is not binding, $b_1 F + b_2 A < Z$.*

Proof. Follows immediately from $v_1 < v_2$ and Eq. (19). □

Lemma 2. *It cannot be optimal that $\mu_1 = \mu_2 = 0$ at the same time, i.e. that capacity is not fully used, $G(F+A) < Q(K)$, and the emission limit is not binding, $b_1 F + b_2 A < Z$.*

Proof. Follows immediately from Eq. (12) and Eq. (18b). □

Now it is convenient to consider three different cases characterized by small, medium, and large capital stocks:

- *Case 1:* $K \leq \bar{K}$.
- *Case 2:* $\bar{K} < \tilde{K}$.
- *Case 3:* $K \geq \tilde{K}$.

4.1. The analysis of case 1: $K \leq \bar{K}$

Clearly, by definition of $\bar{K}$, this implies that for $K < \bar{K}$ the emission limit is not binding, $E < Z$. By Lemmas 1 and 2, this means that $A = 0$ and that full capacity is used. Therefore, in this case F has a one to one correspondence to K, that can be denoted as $F = F(K)$, which satisfies

$$G(F(K)) = Q(K)$$
$$\Rightarrow F(K) = G^{-1}(Q(K)) = S(K).$$

Fig. 1 illustrates this situation.

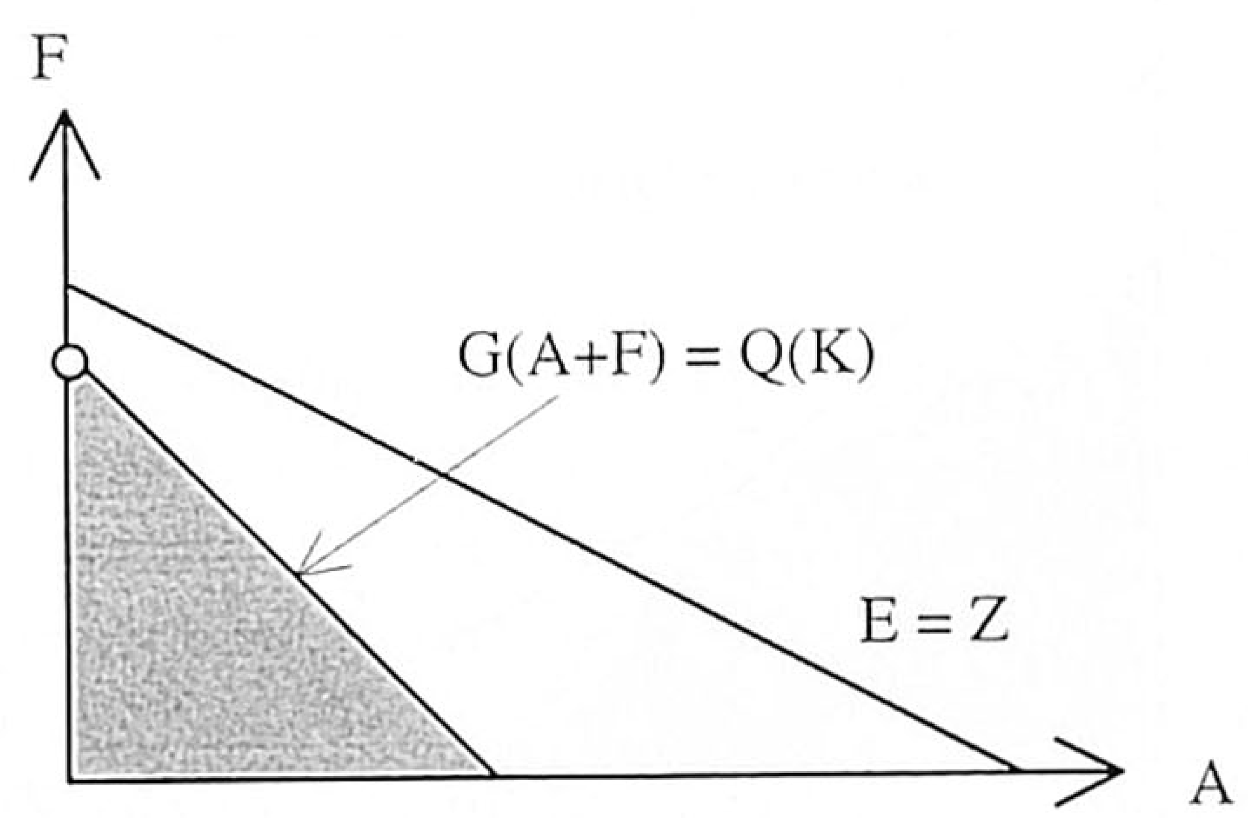

Fig. 1. The case $K \leq \bar{K}$.

4.2. *The analysis of case 2: $\bar{K} < \bar{K}$*

Clearly, since $K > \bar{K}$ this means that for $A = 0$ the emission limit is violated when using full capacity. Since $K < \tilde{K}$, using full capacity while producing with only the clean input, A, (i.e., $F = 0$) causes emissions being less than Z. Thus there is an intersection of the lines $G(A + F) = Q(K)$ and $E = Z$ in the A-F-plane; see Fig. 2.

From Lemma 1 we know that $E = Z$ if $A > 0$, and since Lemma 2 implies that $E = Z$ if $G(F + A) < Q(K)$, we can conclude that only the following subcases 2.1, 2.2 and 2.3 are possible; see also Fig. 2.

4.2.1. *Subcase 2.1: $A > 0$, $F > 0$, $q < Q(K)$*

By Eqs. (18c,e), $\mu_3 = \mu_4 = 0$ and $\mu_1 = 0$. Now, Eqs. (18a,b) yield

$$pG'(F + A) = \frac{v_2 b_1 - v_1 b_2}{b_1 - b_2} > 1. \tag{20}$$

Since G is strictly concave, F and A are uniquely determined by Eq. (20) and the emissions being equal to their upper bound:

$$E = b_1 F + b_2 A = Z.$$

Furthermore, Eq. (20) can also be written as

$$\frac{pG'(F + A) - v_1}{b_1} = \frac{pG'(F + A) - v_2}{b_2}. \tag{21}$$

This implies that in this subcase the additional cash flow per unit of additional emissions, resulting from producing with an extra unit of the variable input, is the same for both inputs.

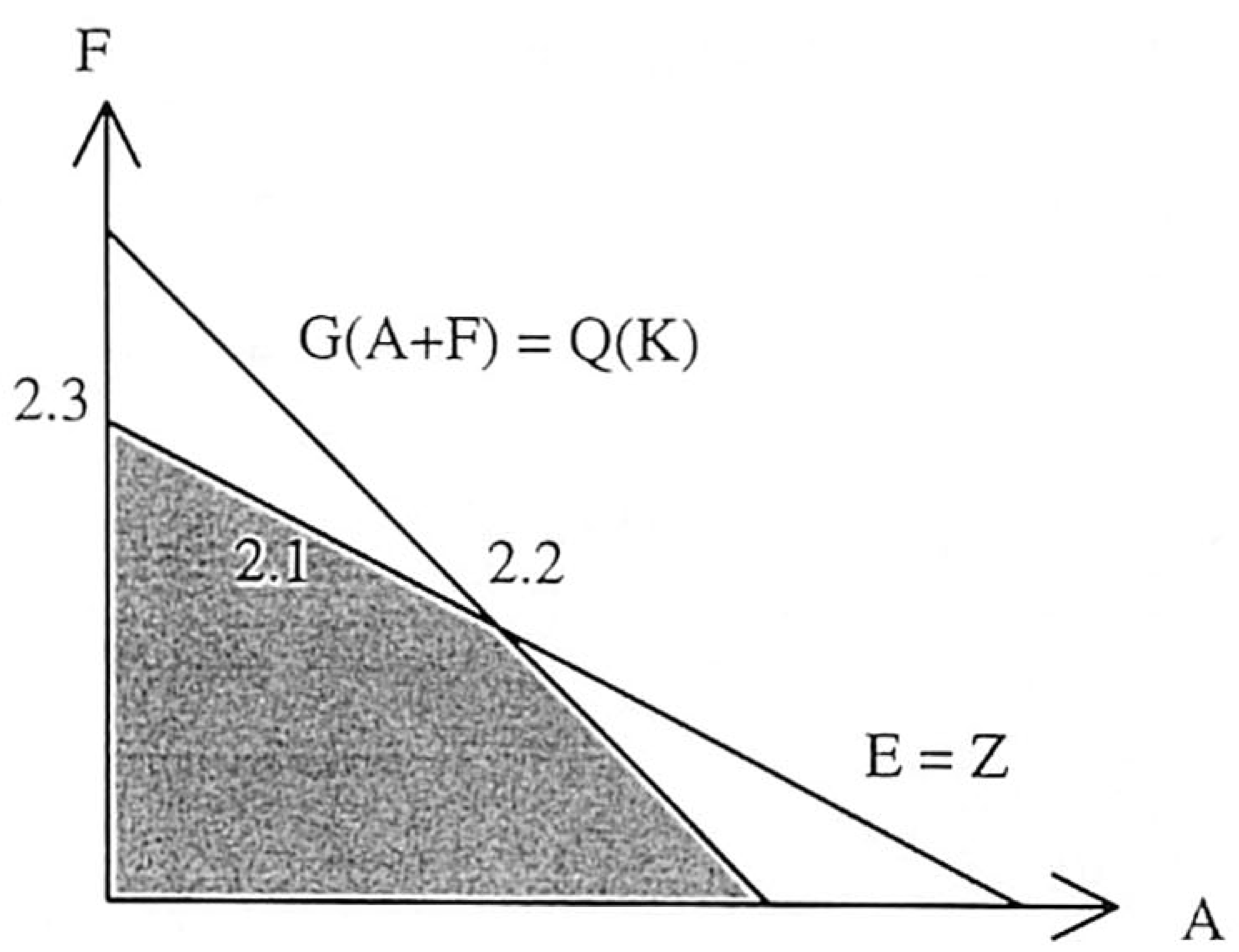

Fig. 2. The case $\bar{K} < K < \tilde{K}$.

4.2.2. *Subcase 2.2: $A > 0$, $F > 0$, $q = Q(K)$*

By Eq. (18e), $\mu_3 = \mu_4 = 0$. Using Eqs. (18a,b), straightforward calculations show that

$$pG'(F + A) > \frac{b_1 v_2 - b_2 v_1}{b_1 - b_2}. \tag{22}$$

Writing this as

$$\frac{pG'(F + A) - v_1}{b_1} < \frac{pG'(F + A) - v_2}{b_2},$$

we conclude that in this case the additional cash flow per unit of additional emissions resulting from producing with an extra unit of A exceeds the additional cash flow per unit of additional emissions resulting from producing with an extra unit of F. Therefore, it makes sense that in this subcase the firm chooses to reduce emissions by using the clean input A rather than by leaving a part of the production capacity unused.

Therefore, capacity is used at the maximal level while the emission constraint is binding.

F and A are uniquely determined by

$$Q(K) = G(F + A), \qquad Z = b_1 F + b_2 A. \tag{23}$$

4.2.3. *Subcase 2.3: $A = 0$, $F > 0$, $q < Q(K)$*

By Eqs. (18c,e), $\mu_4 = 0$ and $\mu_1 = 0$. Therefore, Eqs. (18a,b) yield

$$\mu_2 = \frac{pG'(F) - v_1}{b_1} > \frac{pG'(F) - v_2}{b_2},$$

which implies that the additional cash flow per unit of additional emissions arising from producing with an extra unit of F exceeds the additional cash flow per unit of additional emissions due to production with an extra unit of A. This explains why the expensive clean input is not used in this subcase. The above inequality leads to

$$pG'(F) < \frac{b_1 v_2 - b_2 v_1}{b_1 - b_2}. \tag{24}$$

Notice that the firm uses F while the emission constraint is binding, so that $F = Z/b_1 = \bar{F}$.

4.3. *The analysis of case 3: $K \geq \tilde{K}$*

Clearly, if $K > \tilde{K}$, this means that full capacity can never be used even if the firm produces with

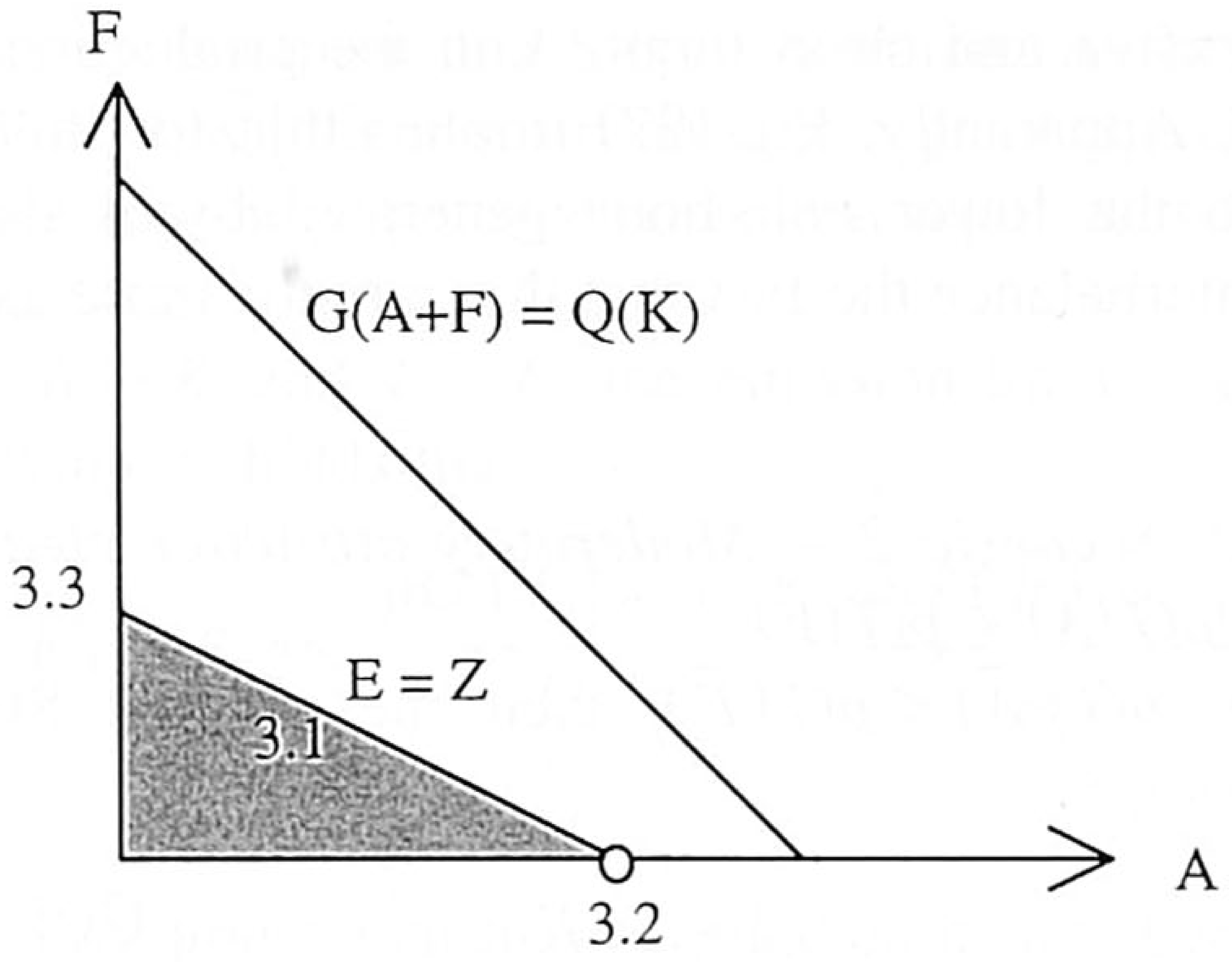

Fig. 3. The case $K \geq \tilde{K}$.

only the clean input, i.e. we have the situation depicted in Fig. 3, where the capacity constraint is never binding.

Lemma 2 implies that the emission limit must be binding.

4.3.1. Subcase 3.1: $A > 0$, $F > 0$, $q < Q(K)$

The equations for determining F and A and the conditions under which this subcase is optimal are the same as in subcase 2.1.

4.3.2. Subcase 3.2: $A > 0$, $F = 0$, $q < Q(K)$

By Eqs. (18c,e), $\mu_3 = 0$ and $\mu_1 = 0$. Then, Eqs. (18a,b) imply

$$\frac{pG'(A) - v_2}{b_2} > \frac{pG'(A) - v_1}{b_1}.$$

Hence, the firm does not produce with F, since the marginal cash flow per unit of marginal emissions, resulting from producing with an extra unit of F, is less than the marginal cash flow per unit of marginal emissions, resulting from producing with an extra unit of A.

The above inequality can be written as

$$pG'(A) > \frac{b_1 v_2 - b_2 v_1}{b_1 - b_2}. \tag{25}$$

Here we see that the emission constraint is binding while the firm only uses A. This implies that $A = Z/b_2 = \tilde{A}$.

4.3.3. Subcase 3.3: $A = 0$, $F > 0$, $q < Q(K)$

The equations for determining F and A and the conditions under which this subcase is optimal are the same as in as subcase 2.3.

4.4. Summary of the results of Step 1 – Different scenarios

Let us define

$$B = \frac{b_1 v_2 - b_2 v_1}{b_1 - b_2} > 1. \tag{26}$$

From the above cases and their analysis we obtain the following three possible scenarios ($\bar{F}$ and $\tilde{A}$ are defined in Eq. (14) and Eq. (15)):

- *Scenario 1: $pG'(\bar{F}) < B$.*
- *Scenario 2: $pG'(\tilde{A}) < pG'(\bar{F})$.*
- *Scenario 3: $pG'(\tilde{A}) > B$.*

To interpret Scenario 1, notice that $pG'(\bar{F}) < B$ can be rewritten into

$$\frac{pG'(\bar{F}) - v_2}{b_2} < \frac{pG'(\bar{F}) - v_1}{b_1}. \tag{27}$$

Since G is concave and $b_1 > b_2$, this inequality also holds for input levels exceeding $\bar{F}$. This implies that for all these levels it holds that the marginal cash flow per unit of marginal emissions due to producing with an extra unit of input is greater for F than for A. Therefore, in what follows we denote Scenario 1 by 'Unattractive Clean Input Scenario'.

In Scenario 3 we have $pG'(\tilde{A}) > B$. Since G is concave it holds for all input levels $F + A$ below $\tilde{A}$ that

$$pG'(F + A) > B$$
$$\Rightarrow \frac{pG'(F + A) - v_2}{b_2} > \frac{pG'(F + A) - v_1}{b_1}. \tag{28}$$

Hence, producing with an extra input A yields the highest cash flow per unit of restricted emissions. Therefore, we will denote this scenario by 'Attractive Clean Input Scenario'.

For obvious reasons, in what follows the intermediate Scenario 2 will be denoted by 'Moderately Attractive Clean Input Scenario'.

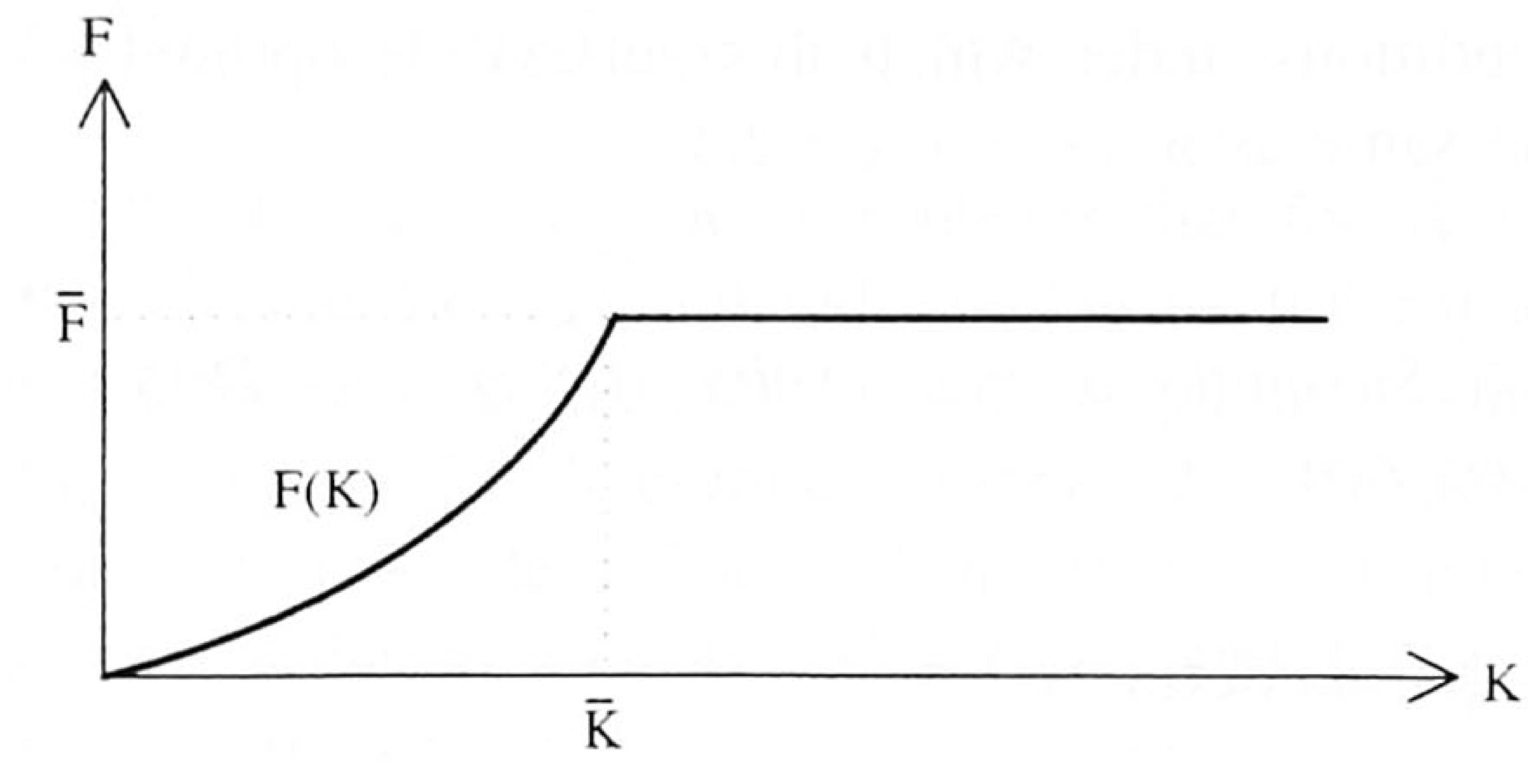

Fig. 4. Solution of Step 1 in the Unattractive Clean Input Scenario 1.

4.4.1. Scenario 1 – Unattractive Clean Input: $pG'(\bar{F}) < B$

If $pG'(\bar{F}) < B$, then the optimal Step 1 solution is

$$A = 0, \qquad F = \begin{cases} S(K), \\ \bar{F}, \end{cases} \qquad q = \begin{cases} Q(K), \\ Q(\bar{K}), \end{cases}$$

$$E\begin{cases} < Z, \\ = Z, \end{cases} \quad \text{for } K \in \begin{cases} [0, \bar{K}], & \text{case 1}, \\ (\bar{K}, \infty), & \text{case 2.3 or 3.3}. \end{cases} \tag{29}$$

Since $S(K)$ is, by assumption, convex (see Section 2), also $F(K)$ will be convex for $K < \bar{K}$. This case is depicted in Fig. 4.

Clearly, if the total amount of input, $F + A$, falls below $\bar{F}$, emissions remain below their upper limit; see Eq. (14). Then there is no need to produce with the expensive input A so that putting A equal to zero is optimal in this case.

Remember that from Eq. (27) we could derive that for total input levels exceeding $\bar{F}$ it holds that marginal cash flow per unit of restricted emissions is greater for F than for A. Hence, we can conclude that here it is never optimal for the firm to use the expensive and clean input A in the production process. Apparently, Eq. (27) implies that for this scenario the lower emissions generated by A do not counterbalance the fact that this input is more expensive.

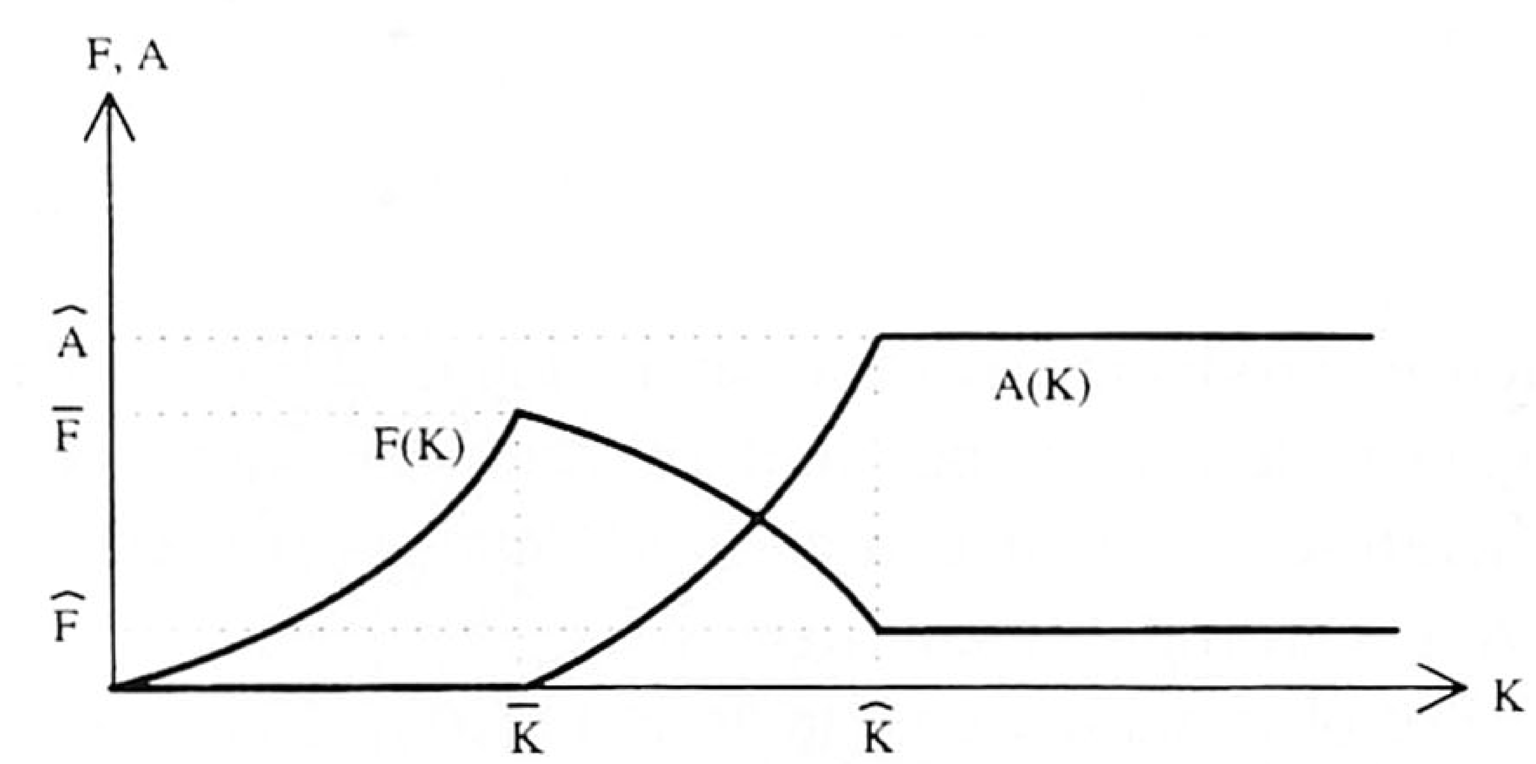

Fig. 5. Solution of Step 1 in the Moderately Attractive Clean Input Scenario 2.

4.4.2. Scenario 2 – Moderately attractive clean input: $pG'(\tilde{A}) < pG'(\bar{F})$

If $pG'(\tilde{A}) < pG'(\bar{F})$, then the optimal Step 1 solution is

$$A\begin{cases} = 0, \\ > 0, \\ = \hat{A} < \tilde{A}, \end{cases} \qquad F\begin{cases} = S(K), \\ > 0, \\ = \hat{F} < \bar{F}, \end{cases} \qquad q = \begin{cases} Q(K), \\ Q(K), \\ Q(\hat{K}), \end{cases}$$

$$E\begin{cases} < Z, \\ = Z, \\ = Z, \end{cases} \quad \text{for } K \in \begin{cases} [0, \bar{K}], & \text{case 1}, \\ (\bar{K}, \hat{K}), & \text{case 2.2}, \\ [\hat{K}, \infty), & \text{case 2.1 or 3.1}, \end{cases} \tag{30}$$

where $\hat{A}$ and $\hat{F}$ are uniquely determined by

$$pG'(F + A) = B,$$

i.e.

$$F + A = (G')^{-1}\left(\frac{B}{p}\right),$$

$$E(A, F) = b_1 F + b_2 A = Z, \tag{31}$$

which yields

$$\hat{A} = \frac{Z}{b_2 - b_1} - \frac{b_1}{b_2 - b_1}(G')^{-1}\left(\frac{B}{p}\right),$$

$$\hat{F} = -\frac{Z}{b_2 - b_1} + \frac{b_2}{b_2 - b_1}(G')^{-1}\left(\frac{B}{p}\right).$$

It is easily checked that the non-negativity of $\hat{A}$ and $\hat{F}$ is equivalent with

$$pG'(\tilde{A}) < pG'(\bar{F}).$$

Furthermore, $\hat{K}$ is defined by

$$G(\hat{A} + \hat{F}) = Q(\hat{K}). \tag{32}$$

The Step 1 solution is presented in Fig. 5; see also Section 4.4.4.

Note that, apart from the fact that $\hat{F} < \bar{F}$, we do not know beforehand how the ranking of the levels of $\bar{F}$, $\hat{F}$, and $\hat{A}$ is.

Let us interpret this Step 1 solution. As before, for $K \in [0, \bar{K}]$ there is no need to replace the cheap input F by the expensive and clean input A, so that A equals zero here.

If $K = \bar{K}$ and $F = \bar{F}$, the emission limit is binding, while it holds that

$$pG'(\bar{F}) > B \Rightarrow \frac{pG'(\bar{F}) - v_2}{b_2} > \frac{pG'(\bar{F}) - v_1}{b_1}. \tag{33}$$

Eq. (33) implies that the marginal cash flow per unit of marginal emissions, due to producing with an additional input, is higher for A than for F. Therefore, it is profitable for the firm to keep on producing with full capacity when K increases marginally, while at the same time the firm must reduce F and increase A such that emissions remain equal to Z. In fact, this policy is optimal for every capital stock level in the interval $(\bar{K}, \hat{K})$.

If, for $K = \hat{K}$, the firm produces with full capacity while the emission limit is binding, it holds that

$$pG'(\hat{F} + \hat{A}) = B$$
$$\Rightarrow \frac{pG'(\hat{F} + \hat{A}) - v_2}{b_2} = \frac{pG'(\hat{F} + \hat{A}) - v_1}{b_1}. \tag{34}$$

This means that the marginal cash flow per unit of marginal emissions, due to producing with an extra input, is the same for both inputs. Therefore, for $K > \hat{K}$ the marginal revenue pG' is too low for counterbalancing the high costs of the expensive input A, when the firm produces with full capacity. This implies that, for stocks of capital goods being larger than $\hat{K}$, it is optimal to leave some capacity

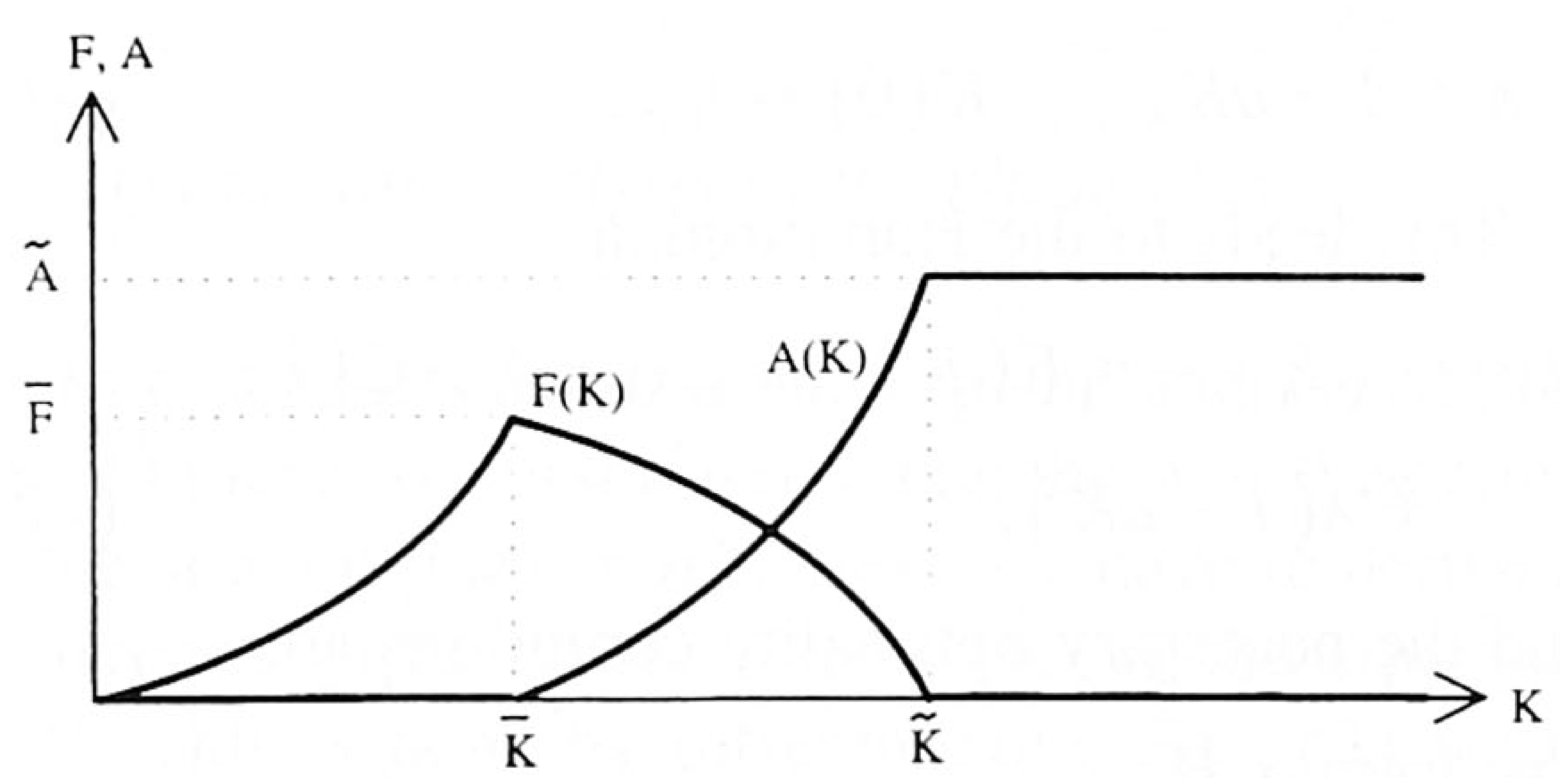

Fig. 6. Solution of Step 1 in the Attractive Clean Input Scenario 3.

unused so that A does not need to be larger than $\hat{A}$ in order to satisfy the emission limit.

4.4.3. Scenario 3 – Attractive clean input: $pG'(\tilde{A}) > B$

If $pG'(\tilde{A}) > B$, then the optimal Step 1 solution is depicted in Fig. 6 and is given by

$$A\begin{cases} = 0, \\ > 0, \\ = \tilde{A}, \end{cases} \quad F\begin{cases} = S(K), \\ > 0, \\ = 0, \end{cases} \quad q = \begin{cases} Q(K), \\ Q(K), \\ Q(\tilde{K}), \end{cases}$$

$$E\begin{cases} < Z, \\ = Z, \\ = Z, \end{cases} \quad \text{for } K \in \begin{cases} [0, \bar{K}], & \text{case 1}, \\ (\bar{K}, \tilde{K}), & \text{case 2.2}, \\ [\tilde{K}, \infty), & \text{case 3.2}. \end{cases} \tag{35}$$

Again, for $K \in [0, \bar{K}]$ the emission limit is automatically satisfied. Therefore, it makes no sense here to leave some capacity unused or to use the expensive input A in the production process.

In case $K \in (\bar{K}, \tilde{K})$ we can derive from Eq. (28) that producing with input A yields the highest cash flow per unit of restricted emissions. Therefore, the firm produces with full capacity while it adjusts the input levels F and A such that the emission constraint is binding.

For $K \in [\tilde{K}, \infty]$ it holds that producing with full capacity always violates the emission constraint, even when only the clean input A is used. Therefore, the firm is forced to leave some capacity unused. It keeps emissions equal to Z by fixing the inputs F and A at zero and $\tilde{A}$, respectively.

We note that the different cases and the results of Figs. 4–6 can also be depicted in three-dimensional pictures in (K, F, A)-space. Due to space restrictions these are omitted here and can be found in a working paper which can be obtained from the authors.

4.4.4. Variable inputs as functions of capital stock under full capacity and binding standard

In Scenarios 2 and 3, i.e., in the Moderately Attractive Clean Input Scenario and the Attractive Clean Input Scenario, the functions $A(K)$ and $F(K)$ appeared in subcase 2.2. Given the objective (17a), it is important to investigate whether the function $v_1 F(K) + v_2 A(K)$ is convex or not. We recall from

Eq. (23) that $A(K)$ and $F(K)$ are defined by

$$F + A = S(K) \text{ and } b_1 F + b_2 A = Z, \tag{36}$$

where, as already stated in Section 2, $S(K)$ is convex and defined as

$$S(K) = G^{-1}(Q(K)). \tag{37}$$

The solution of this system of linear equations is

$$A(K) = \frac{b_1}{b_1 - b_2} S(K) - \frac{Z}{b_1 - b_2},$$

$$F(K) = \frac{-b_2}{b_1 - b_2} S(K) + \frac{Z}{b_1 - b_2}. \tag{38}$$

Thus, for the Moderately Attractive Clean Input and the Attractive Clean Input Scenarios we obtain the following result:

Lemma 3. *In subcase* 2.2 *the cost of input is given by*

$$v_1 F(K) + v_2 A(K) = B \cdot S(K) - \frac{v_2 - v_1}{b_1 - b_2} Z, \tag{39}$$

which is convex, since $S(K)$ *is convex.*

In order to compare this input cost with the corresponding term in case 1, we know that for this case it holds that $F = S(K)$ and $A = 0$, i.e., the corresponding input cost in case 1 is $v_1 S(K)$. It is

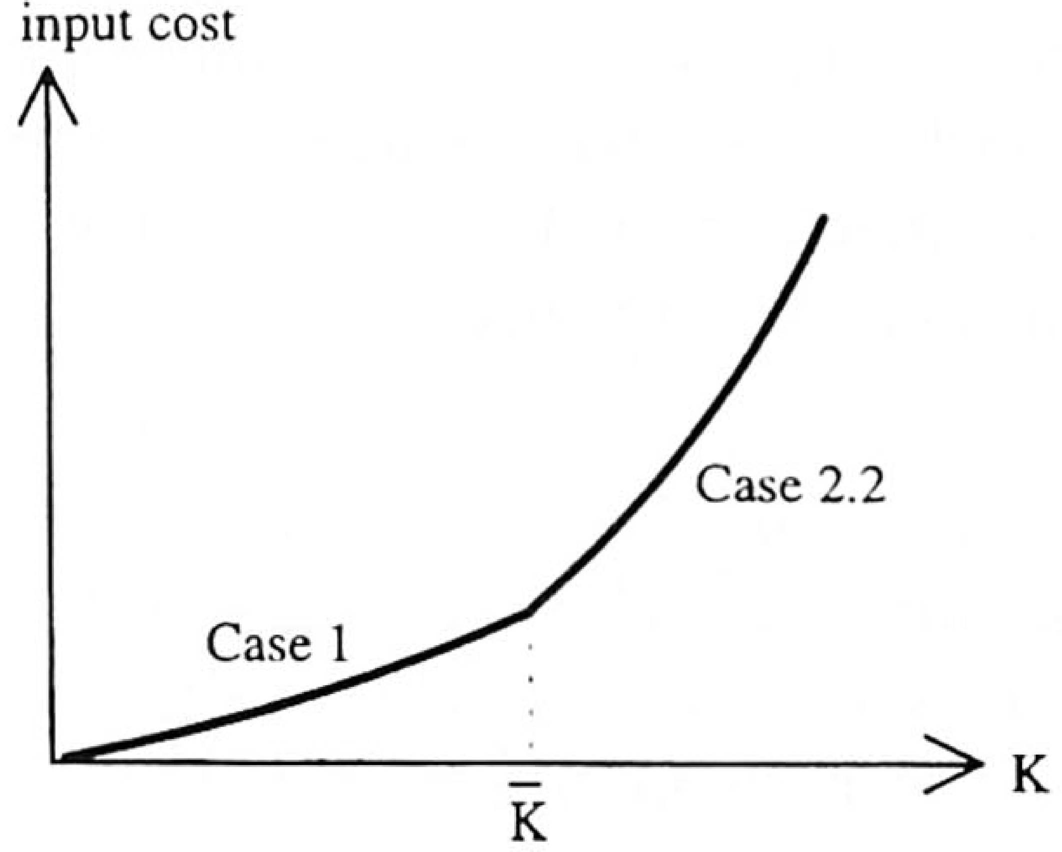

Fig. 7. The cost of variable input $v_1 F + v_2 A$ in the Moderately Attractive Clean Input Scenario and the Attractive Clean Input Scenario is a convex function of capital for $K < \hat{K}$ and $K < \tilde{K}$, respectively.

easily checked that for $K = \bar{K}$ this term yields the same result as Eq. (39).

Furthermore, from $B > v_1$ it is clear that the following lemma holds:

Lemma 4. *As K increases in the Moderately Attractive Clean Input and the Attractive Clean Input Scenario one passes from case* 1 *to subcase* 2.2 *at* $K = \bar{K}$. *In this point, the marginal input cost jumps upwards from* $v_1 S'(K)$ *to* $BS'(K)$.

From an economic point of view, Lemma 4 is easy to understand, since, contrary to the case where $K < \bar{K}$, for $K > \bar{K}$ the firm must use the expensive input A to satisfy the emission limit when the capital stock increases marginally.

Fig. 7 illustrates Lemma 3 and 4.

5. Analysis of Step 2

We only consider the Attractive Clean Input Scenario, because the analysis for the other two scenarios is very similar. Nevertheless, a detailed analysis of these scenarios is available from the authors upon request.

5.1. The maximum principle

With $q(K)$, $A(K)$ and $F(K)$ computed in Step 1, we now solve the following control problem:

$$\max_I \int_0^\infty e^{-rt} \{ pq(K) - v_1 F(K) - v_2 A(K) - C(I) \} \mathrm{d}t \tag{40}$$

s.t.

$$\dot{K} = I - aK, \qquad K(0) = K_0. \tag{41}$$

This leads to the Hamiltonian

$$H = pq(K) - v_1 F(K) - v_2 A(K) - C(I) + \lambda(I - aK), \tag{42}$$

and the necessary optimality conditions are

$$I = \underset{I}{\operatorname{argmax}}\, H,$$

i.e.,

$$\lambda = C'(I), \tag{43}$$

$$\dot{\lambda} = r\lambda - H_K = (r+a)\lambda - pq'(K) + v_1 F'(K) + v_2 A'(K). \tag{44}$$

Strictly speaking, the adjoint equation (44) only holds for those K where $F(K)$, $A(K)$ and $q(K)$ are differentiable in K. At the kinks $K = \bar{K}$ and $K = \tilde{K}$, the differential equation must be replaced by the differential inclusion

$$r\lambda - \dot{\lambda} \in \partial_K H, \tag{45}$$

where ∂_K denotes the generalized gradient of H w.r.t. K; see Clarke (1983). In our case, Eq. (45) evaluated at the kinks mentioned above is the interval between r.h.s. and l.h.s. derivative of H w.r.t. K. In Appendix A these intervals are investigated and it is shown that all intervals are non-empty, which means that existence of the solution is assured.

Usually, in infinite horizon problems it is difficult to verify that the limiting transversality condition

$$\lim_{t\to\infty} \mathrm{e}^{-rt}\lambda(t) = 0$$

is part of the necessary conditions. In this case, however, we know from the results of Step 1 and the fact that $S'(\infty) < \infty$, that

$$\frac{\partial}{\partial K}(pq(K) - v_1 F(K) - v_2 A(K) - C(I))$$

is bounded so that the growth conditions imposed by Seierstad (1977) are satisfied. This theorem then guarantees that the optimal solution satisfies this transversality condition. Hence, we will only look for those solutions of the canonical system for which K and λ remain bounded.

5.2. *Qualitative analysis in the phase plane*

In order to perform a phase plane analysis in the (K, I)-plane, we first observe that the $\dot{K} = 0$ isocline is the straight line $I = aK$. Next, we have to derive a differential equation for I from the adjoint equation (44). This is done by differentiating Eq. (43) w.r.t. time t and subsequent elimination of the costate λ using Eq. (43). Keeping in mind the kinks mentioned above, this yields

$$\dot{I} = \frac{1}{C''(I)}[(r+a)C'(I) - pQ'(K) + v_1 S'(K)] \quad \text{for } K < \bar{K} \tag{46a}$$

and

$$\dot{I} = \frac{1}{C''(I)}[(r+a)C'(I) - pQ'(K) + BS'(K)] \quad \text{for } \bar{K} < \tilde{K} \tag{46b}$$

(see also Lemma 3), and finally

$$\dot{I} = \frac{1}{C''(I)}[(r+a)C'(I)] \quad \text{for } K > \tilde{K}. \tag{46c}$$

With these, we can compute the slope of the $\dot{I} = 0$ isocline. It is convenient to treat the three regions separately.

First, we consider the region of a small capital stock where the emission constraint is not binding, i.e., $K < \bar{K}$. In this case we obtain

$$\left.\frac{\mathrm{d}I}{\mathrm{d}K}\right|_{\dot{I}=0} = \frac{pQ''(K) - v_1 S''(K)}{(r+a)C''(I)} < 0, \tag{47a}$$

since we have assumed that $S(K)$ is convex.

On the other hand, when the capital stock is so large that the emission constraint is binding but still $q = Q(K)$, i.e., for $\bar{K} < \tilde{K}$, we obtain

$$\left.\frac{\mathrm{d}I}{\mathrm{d}K}\right|_{\dot{I}=0} = \frac{pQ'' - BS''}{(r+a)C''} < 0. \tag{47b}$$

Finally, when the capital stock is so large that it is not optimal to use the total production capacity, i.e., in the region $K > \tilde{K}$, then Eq. (46c) holds and, due to Eq. (10), we can conclude that $\dot{I} > 0$ in this case.

Let us now consider a steady state (K^*, I^*), which is defined by

$$I^* = aK^*,$$

$$pQ'(K^*) = (r+a)C'(I^*) + v_1 S'(K^*) \tag{48}$$

if $K^* < \bar{K}$; see case (a) in Section 5.4.

The second equation in (48) says that in equilibrium marginal revenue equals marginal investment costs plus marginal input costs.

The determinant of the Jacobian of the dynamical system (41) and (46a) evaluated at the equilibrium can easily be computed:

$$\det J = \frac{1}{C''}\left[-a(r+a)C'' + pQ'' - v_1 S''\right] < 0. \tag{49}$$

This means the following proposition holds:

Proposition 2. *If $K^* < \bar{K}$, then the equilibrium is a saddle point.*

On the other hand, for $\bar{K} < \tilde{K}$ the steady state is defined by

$$I^* = aK^*,$$
$$pQ'(K^*) = (r+a)C'(I^*) + BS'(K^*) \tag{50}$$

(see case (c) in Section 5.4).

Here, opposite to the case $K^* < \bar{K}$, additional expenses on input (more expensive input A) are necessary to keep pollution equal to the standard level when capital stock increases marginally. Consequently, marginal expenses are $BS'(K^*)$ which are higher than marginal input expenses $v_1 S'(K^*)$ in the unregulated case. This results in a smaller equilibrium compared to the unregulated case.

Now, the dynamical system consists of Eqs. (41) and (46b), implying that the determinant of the Jacobian, evaluated at the steady state, equals

$$\det J = \frac{1}{C''}\left[-a(r+a)C'' + pQ'' - BS''\right] < 0. \tag{51}$$

This yields the following proposition:

Proposition 3. *The equilibrium (K^*, I^*) is a saddle point if $\bar{K} < K^* < \tilde{K}$.*

We note that for $K > \tilde{K}$ no equilibrium is possible, since the $\dot{I} = 0$ isocline does not exist ($\dot{I}$ is always positive).

5.3. Optimal firm behavior without environmental regulation

Before we proceed with the phase plane analysis, let us first derive the solution in the case of no regulation, so that we have a benchmark case. Then, there is no incentive for using the expensive clean input or leaving capacity unused. This leads to a phase plane where the $\dot{I} = 0$ isocline and the $\dot{K} = 0$ isocline can be obtained from Eq. (46a) and Eq. (41) respectively (see Fig. 8).

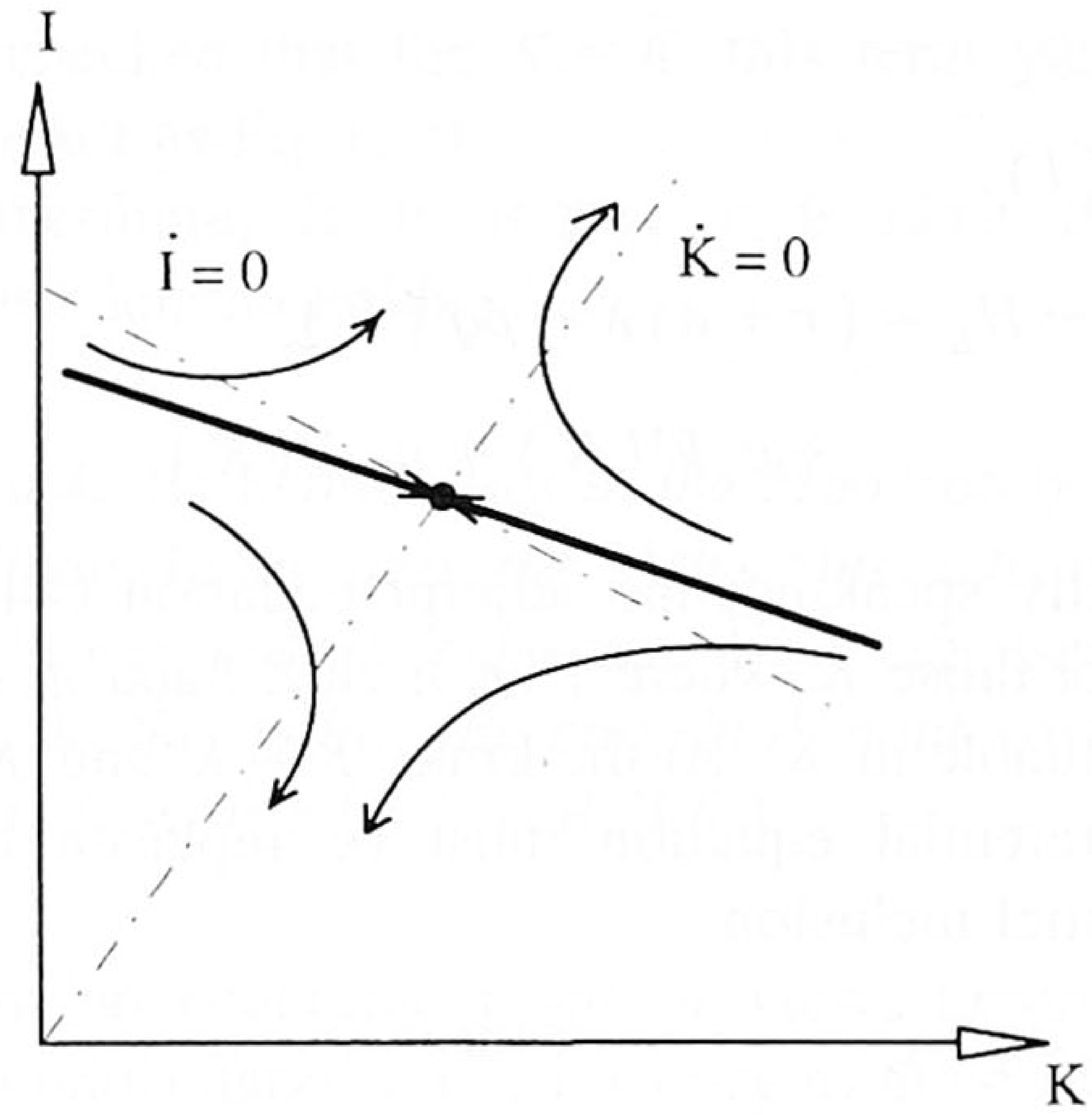

Fig. 8. The benchmark case of no environmental regulation.

The steady state is presented in Eq. (48) and, due to Eq. (49), we know that it is a saddle point. After solving the differential equation (44) (with $F(K) = S(K)$, $A(K) = 0$ and $q(K) = Q(K)$), substituting Eq. (43) into this relation and using Eq. (48) as a fixed point, we derive the following condition which holds for each t:

$$\int_t^{\infty} e^{-(r+a)(s-t)}\{pQ'(K(s)) - v_1 S'(K(s))\}ds - C'(I(t)) = 0, \tag{52}$$

where the left-hand side is the net present value of marginal investment. For an interpretation consider the acquisition of an extra unit of capital at time t. The firm incurs an extra expense at time t in amount of C'. On the other hand, the marginal unit of capital generates – as of time t – a stream of cash flow. This cash flow stream, that is corrected for depreciation and discounting, equals revenue from selling products made with the extra capital stock minus input spendings that are needed for this extra production. Condition (52) then states that the net present value of marginal investment equals zero. Hence, the fundamental economic principle of balancing marginal revenue with marginal expenses applies.

5.4. Qualitative analysis of the solution in the phase plane

From Section 5.2 we know that the $I = 0$ isocline is always decreasing. It is clear from Lemma 4, Eq. (46a), and Eq. (46b) that the $\dot{I} = 0$ isocline suffers a downward jump at $K = \tilde{K}$. The $I = 0$ isocline is vertically shaped at $K = \tilde{K}$, since, by Eq. (46c), it always holds that $\dot{I} > 0$ for $K > \tilde{K}$. Therefore we have four possibilities concerning the intersection of the two isoclines:

(a) intersection for $K < \overline{K}$, i.e. where constraint (9) is not binding;

(b) intersection for $K = \overline{K}$, i.e. at the first discontinuity of the $\dot{I}\} = 0$ isocline;

(c) intersection for $\overline{K} < \tilde{K}$, i.e. where constraint (9) is binding;

(d) intersection for $K = \tilde{K}$.

Additional assumption (13) guarantees that always one of these intersections will occur.

The possible phase diagrams are illustrated in Fig. 9. Cases (a) and (c) refer to ordinary saddle points while the equilibria in cases (b) and (d) are approached within finite time; see, e.g. Feichtinger and Hartl (1986, pp.397–402).

Since Eqs. (14) and (15) imply that the discontinuities in the $\dot{I} = 0$ isocline move to the left as Z decreases, the emission limit is mild in case (a) and becomes more and more tighter in cases (b), (c), and (d), respectively. Therefore, in what follows the cases

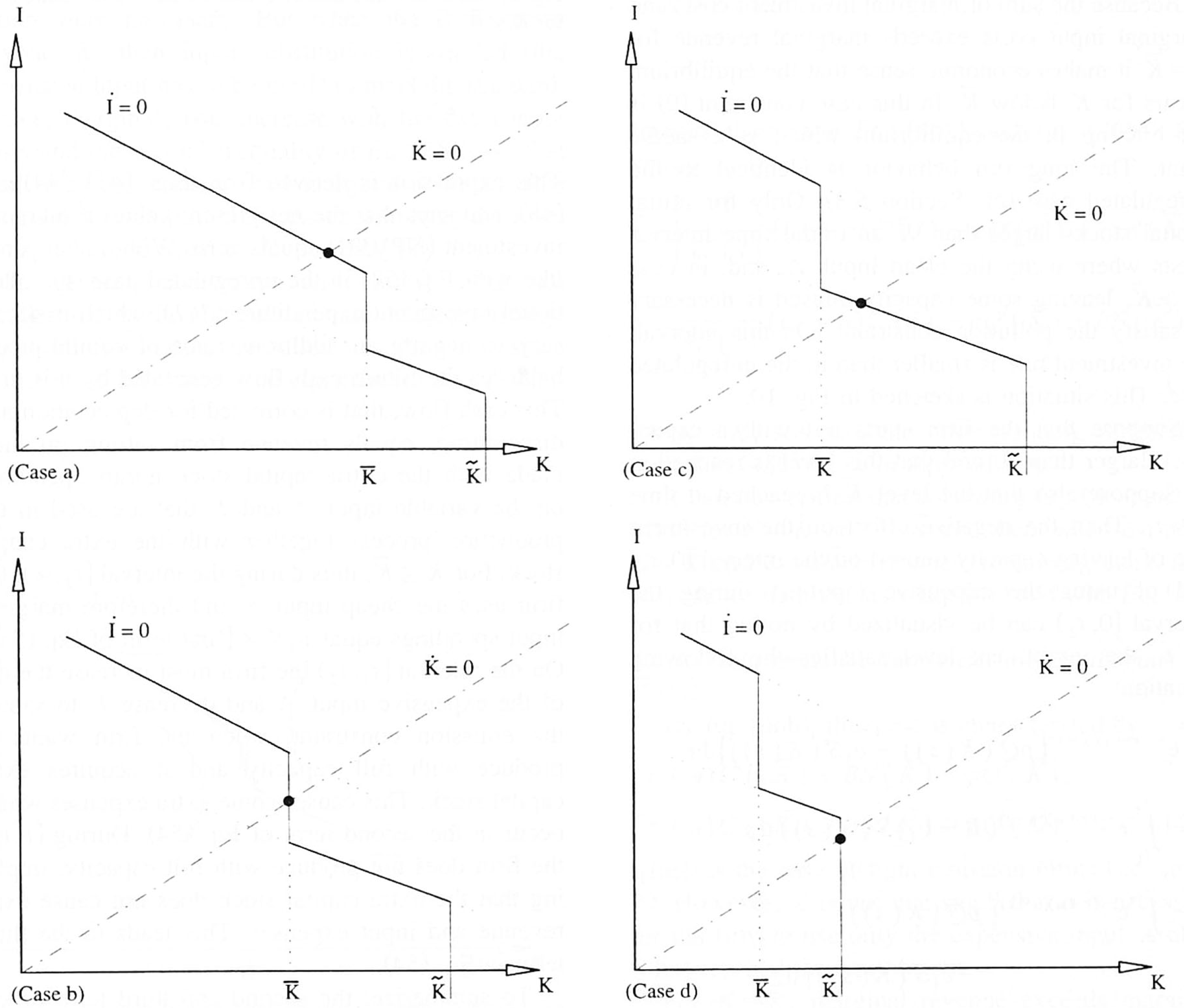

Fig. 9. The four cases in the Attractive Clean Input Scenario.

(a), (b), (c), and (d) will be denoted by *Mild Emission Limit Case*, *Intermediate Emission Limit Case*, *Tight Emission Limit Case*, and *Very Tight Emission Limit Case*, respectively. It is convenient to treat these four cases separately.

5.4.1. Mild emission limit case (a): equilibrium for $K < \bar{K}$:

By Eq. (46a), this case is characterized by

$$(r+a)C'(a\bar{K}) + v_1 S'(\bar{K}) > pQ'(\bar{K}). \tag{53}$$

Clearly, since C' and S' are increasing and Q' is decreasing, this case corresponds to the situation of mild emission limits (i.e. large Z) for fixed parameters r, a, p, and v_1. Note that, by Eq. (14), there is a monotonically increasing relation between $\bar{K}$ and Z.

Because the sum of marginal investment costs and marginal input costs exceeds marginal revenue for $K = \bar{K}$ it makes economic sense that the equilibrium occurs for K below $\bar{K}$. In this case constraint (9) is not binding in the equilibrium which is a saddle point. The long run behavior is identical to the unregulated case (cf. Section 5.3). Only for initial capital stocks larger than $\bar{K}$ an initial time interval exists where using the clean input A, and, in case $K_0 > \tilde{K}$, leaving some capacity unused is necessary to satisfy the pollution constraint. On this interval, the investment rate is smaller than in the unregulated case. This situation is sketched in Fig. 10.

Suppose that the firm starts out with a capital stock larger than $\tilde{K}$ and that this level is reached at t_1. Suppose also that the level $\bar{K}$ is reached at time $t_2 > t_1$. Then the negative effect on the investment rate of leaving capacity unused on the interval $[0, t_1)$ and of using the expensive input A during the interval $[0, t_2)$ can be visualized by noting that for $t < t_1$ the investment level satisfies the following equation:

$$\begin{aligned}&\int_t^{\infty} e^{-(r+a)(s-t)}\{pQ'(K(s)) - v_1 S'(K(s))\}ds\\ &\quad-\int_{t_1}^{t_2} e^{-(r+a)(s-t)}(B - v_1)S'(K(s))\,ds\\ &\quad-\int_t^{t_1} e^{-(r+a)(s-t)}\{pQ'(K(s))\\ &\qquad\qquad -v_1 S'(K(s))\}ds\\ &\quad-C'(I(t)) = 0.\end{aligned} \tag{54}$$

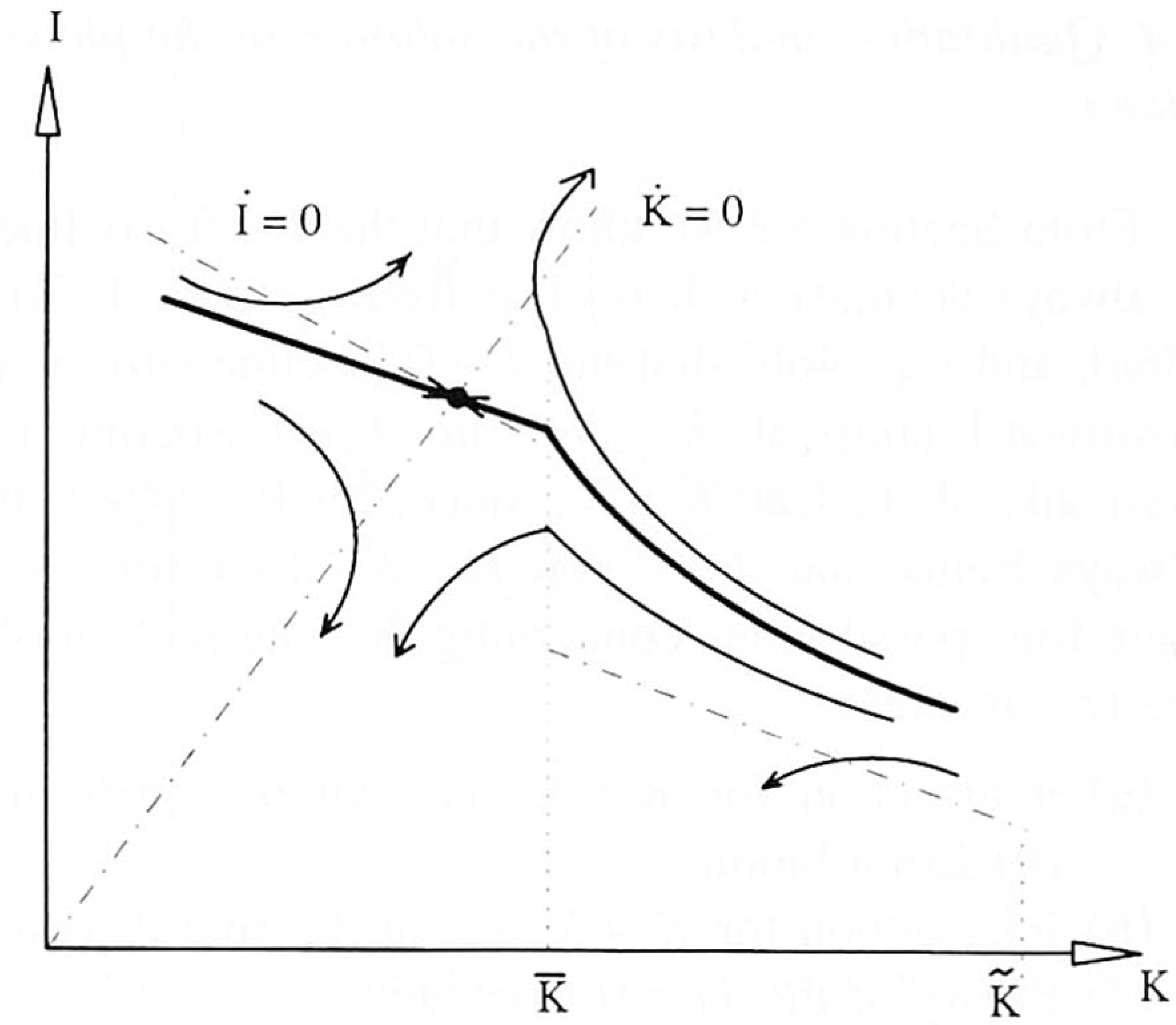

Fig. 10. Case (a): Mild Emission Limit Case with a saddle point for $K < \bar{K}$.

This expression is derived from Eqs. (43), (44) and (48), and says that the net present value of marginal investment (NPVMI) equals zero. With other words, like with Eq. (52) in the unregulated case, the additional investment expenditure $C'(I)$, which is necessary to acquire an additional unit of capital stock, balances the future cash flow generated by this unit. This cash flow, that is corrected for depreciation and discounting, equals revenue from selling products made with the extra capital stock minus spendings on the variable input A and F that are used in the production process together with the extra capital stock. For $K \le \bar{K}$, thus during the interval $[t_2, \infty)$, the firm uses the cheap input F and therefore marginal input spendings equal $v_1 S' \times$ [first term of Eq. (54)]. On the interval $[t_1, t_2)$ the firm must increase the use of the expensive input A and decrease F to satisfy the emission constraint, when the firm wants to produce with full capacity and it acquires extra capital stock. This causes some extra expenses which occur in the second term of Eq. (54). During $[t, t_1)$, the firm does not produce with full capacity, implying that the extra capital stock does not cause extra revenue and input expenses. This leads to the third term in Eq. (54).

To summarize: the second and third term reflect the negative effects that input substitution and unused capacity have on the NPVMI, respectively, and thus on the firm's investment rate.

5.4.2. *Intermediate emission limit case (b): Equilibrium for $K = \bar{K}$*

By Eqs. (46a,b), this intermediate case is characterized by

$$(r+a)C'(a\bar{K}) + v_1 S'(\bar{K})$$
$$\le pQ'(\bar{K}) \le (r+a)C'(a\bar{K}) + BS'(\bar{K}), \qquad (55)$$

which is the case of intermediate values of Z.

From the first inequality of Eq. (55) we infer that marginal revenue exceeds marginal investment costs plus marginal input costs of the cheap input; so it would be optimal to grow further if no input substitution were necessary. But when the firm grows beyond $\bar{K}$, then input substitution is needed (the expensive input has to be used) to meet the standard. Hence, marginal costs increase with the extra input costs and the second inequality of Eq. (55) says that total marginal costs exceed marginal revenue. This means that it is optimal for the firm to keep the level of capital stock equal to $\bar{K}$; see also Fig. 11.

In general, investment and capital stock are smaller than in the unregulated case. Mathematically, the equilibrium cannot be called a saddle point, since the r.h.s. of the differential equation for I is discontinuous there; see Eq. (46a) and Eq. (46b). This yields

$$\dot{I} \in \left[\frac{1}{C''(I)}\left((r+a)C'(I) - pQ'(\bar{K}) + v_1 S'(\bar{K})\right), \frac{1}{C''(I)}\left((r+a)C'(I) - pQ'(\bar{K}) + BS'(\bar{K})\right) \right], \qquad (56)$$

for $K = \bar{K}$; see also the corresponding differential inclusion (45b) for λ in Appendix A.

It can be verified that following Eq. (46a) or Eq. (46b) for $K < \bar{K}$ or $K > \bar{K}$ and then, when the point $K = \bar{K}$ and $I = a\bar{K}$ is reached, remaining there, is in accordance with the state Eq. (41) and with Eq. (56).

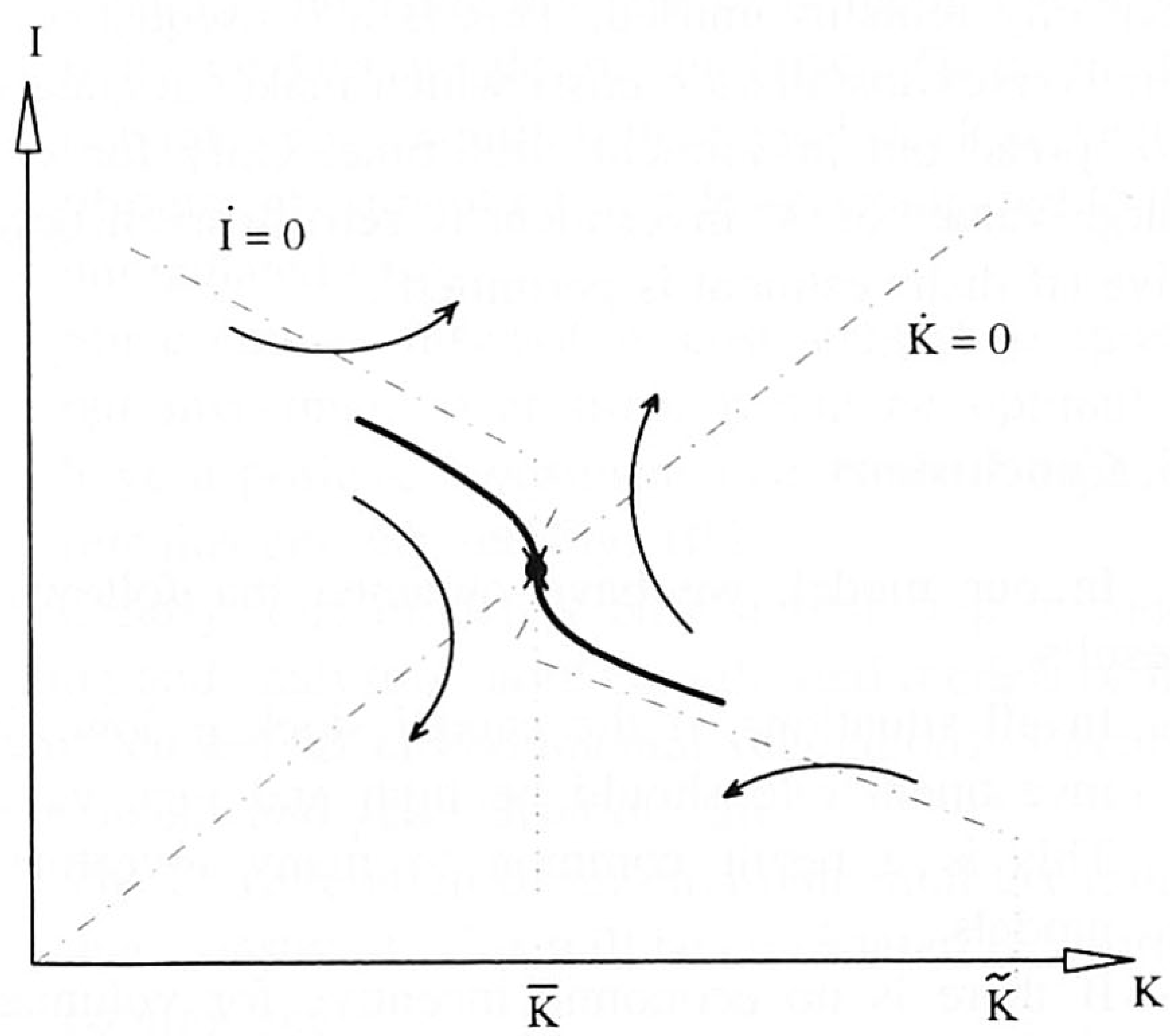

Fig. 11. Case (b): Intermediate Emission Limit Case with an equilibrium for $K = \bar{K}$ which is reached in finite time.

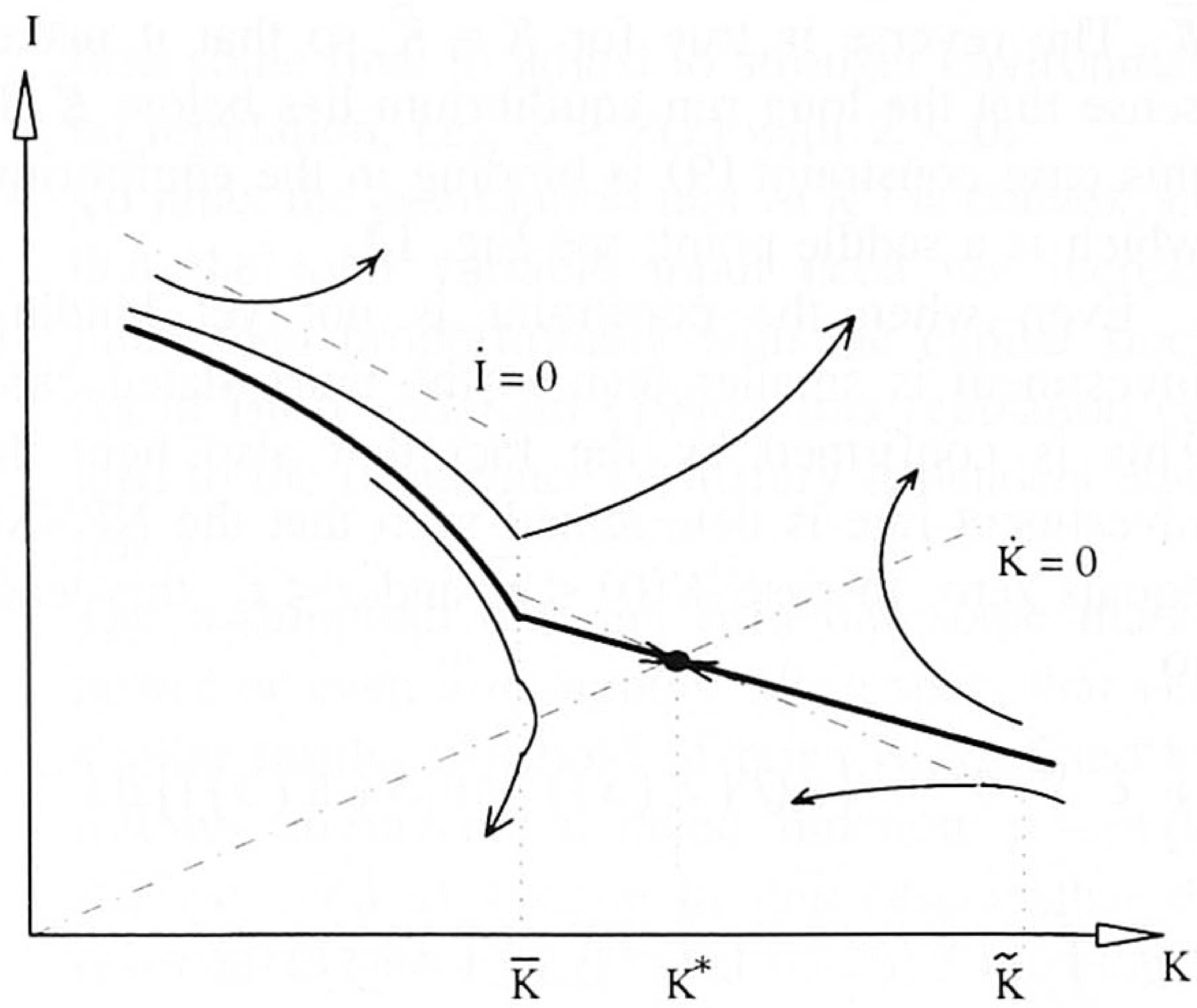

Fig. 12. Case (c): Tight Emission Limit Case with a saddle point for $\bar{K} < K < \tilde{K}$.

5.4.3. *Tight emission limit case (c): Equilibrium for $\bar{K} < K$*

By Eq. (46b), this case is characterized by

$$(r+a)C'(a\bar{K}) + BS'(\bar{K}) < pQ'(\bar{K}),$$
$$(r+a)C'(a\tilde{K}) + BS'(\tilde{K}) > pQ'(\tilde{K}), \qquad (57)$$

which is the case of tight emission limits (i.e. small Z). However, Z is not that small that it is necessary for the firm to use only the expensive input A after it has reached the equilibrium.

For $K = \bar{K}$, marginal revenue exceeds marginal costs, so that it is actually optimal to grow beyond

$\bar{K}$. The reverse is true for $K = \tilde{K}$ so that it makes sense that the long run equilibrium lies below $\tilde{K}$. In this case constraint (9) is binding in the equilibrium which is a saddle point; see Fig. 12.

Even when the constraint is not yet binding, investment is smaller than in the unregulated case. This is confirmed by the fact that also here the investment rate is determined such that the NPVMI equals zero. In case $K(0) \le \bar{K}$ and $t \le t_2'$, this leads to

$$\int_t^{\infty} e^{-(r+a)(s-t)} \{ pQ'(K(s)) - v_1 S'(K(s)) \} \, ds - \int_{t_2'}^{\infty} e^{-(r+a)(s-t)} (B - v_1) S'(K(s)) \, ds - C'(I(t)) = 0, \tag{58}$$

where t_2' denotes the point of time at which the pollution limit becomes binding.

Eq. (58) can be interpreted in the same way as Eq. (54). We can conclude that when it determines its current investment rate, the firm already takes into account the future substitution of the expensive input A for the cheap input F, even when the pollution constraint is not yet binding. Notice that on this trajectory capacity is fully used everywhere.

5.4.4. *Very tight emission limit case (d): Equilibrium for $K = \tilde{K}$*

By Eq. (46b) this case is characterized by

$$(r+a)C'(a\tilde{K}) + BS'(\tilde{K}) \le pQ'(\tilde{K}), \tag{59}$$

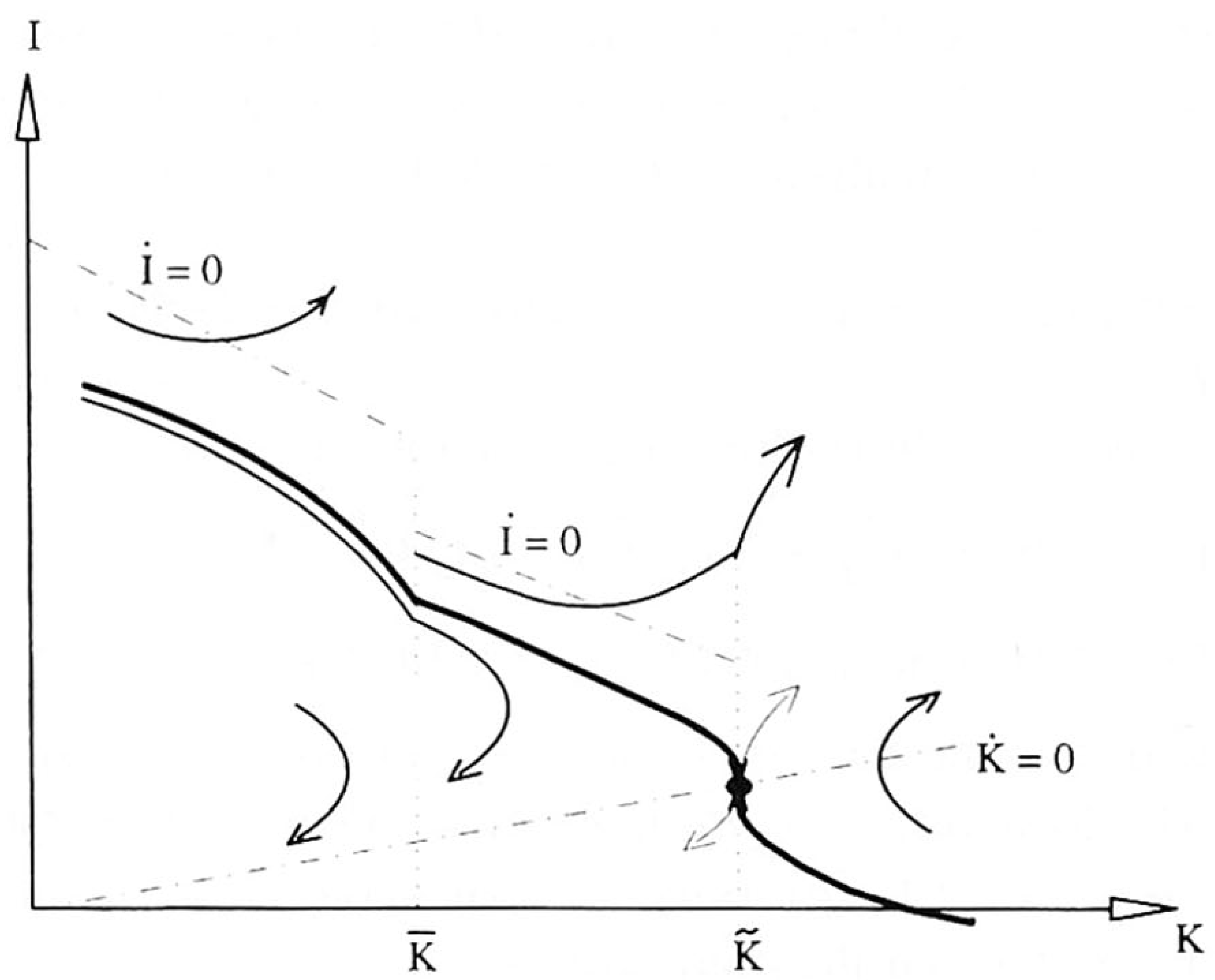

Fig. 13. Case (d): Very Tight Emission Limit Case with equilibrium for $K = \tilde{K}$ which is reached in finite time.

which is the case of very tight emission limits (i.e. very small Z). Inequality (59) implies that marginal revenue exceeds marginal costs, so that it seems to be optimal to grow beyond $\tilde{K}$. But the problem is that here the emission limit is that tight that it will always be violated if the firm produces with full capacity while $K > \tilde{K}$, even when only the clean input A is used. Therefore, the long run equilibrium equals $\tilde{K}$, because it makes no sense to have an equilibrium where it is not allowed to produce with full capacity, which would be the case if $K^* > \tilde{K}$, with K^* denoting the equilibrium capital stock; see Fig. 13.

As in case (b), the equilibrium cannot be called a saddle point, since the r.h.s. of the differential equation for I is discontinuous there; see Eqs. (46b,c). This yields

$$\dot{I} \in \left[\frac{1}{C''(I)} \left((r+a)C'(I) - pQ'(\tilde{K}) + BS'(\tilde{K}) \right), \frac{1}{C''(I)} \left((r+a)C'(I) \right) \right], \tag{60}$$

for $K = \tilde{K}$.

Again, as in case (b), it can be verified that following Eq. (46b) or Eq. (46c) for $\bar{K} < \tilde{K}$ or $K > \tilde{K}$ and then, when the point $K = \tilde{K}$ and $I = a\tilde{K}$ is reached, remaining there, is in accordance with the state equation (41) and with Eq. (60).

It is interesting to note that for $K > \tilde{K}$ it can be optimal to have a positive investment rate, even if capacity remains unused. This is a consequence of the convex installation costs which makes it cheaper to spread out investment over time. Only for very large values of K investment is zero or even negative (if disinvestment is permitted).

6. Conclusions

In our model, we have obtained the following results:

- In all situations, if the capital stock is low, the investment rate should be high and vice versa. This is a result common to many investment models.
- If there is no economic incentive for voluntary substitution of the clean input for the dirty input, the firm will not start the input substitution before

the emission constraint is hit. This is an argument for the introduction of an emission tax which, in general, affects the firms' behavior for all levels of emission.

- If the marginal cash flow per unit of emissions is higher for the dirty input than for the clean input, it can be optimal to leave some production capacity unused rather than to use the expensive clean input in the production process, in order to satisfy the emission constraint.
- During the whole planning period the firm's investment level is determined such that marginal investment expenditures balance the future cash flow stream due to this extra unit of investment. This means that the net present value of marginal investment equals zero. From this we could determine the precise effects on the growth of the firm of both, input substitution and leaving some capacity unused; see, e.g., Eq. (54). Of course, both effects are negative. It turns out that when the firm determines its current investment rate, it already takes into account future input substitution, even when the pollution constraint is not yet binding.
- It is always optimal to approach a long run optimal level of capital. In some cases, this equilibrium is reached within finite time, but usually it will be approached asymptotically.
- Consider the situation where equilibrium capital stock is that large that, when producing with full capacity and using only the dirty input, the pollution standard would be violated. Then, in the regulated case, equilibrium capital stock and equilibrium investment rate are lower compared to the unregulated case.
- Since convex installation cost suggest to spread out investment over time, it can be optimal to have a positive investment rate, even if capacity remains unused; see Fig. 10d.

Clearly, this model is only a first step in modelling and analysing more complicated models of the firm subject to environmental regulations. Possible extensions and generalisations are:

- The consideration of an environmental tax rather than constraint. This will be considered in a forthcoming paper.
- To study a scenario where the pollution standard becomes tighter over time in order to give the firm some time to adjust to stronger environmental regulation, i.e., $Z = Z(t)$ with $\dot{Z} < 0$.
- To relax the assumption that $S(K)$ is convex, i.e. that the total variable input need not increase more than proportionally with the capital stock. As in Hartl and Kort (1996), this relaxation can lead to the occurrence of history dependent equilibria.
- The assumption that the firm has some market power or even a monopoly. We expect that very similar results will hold, if price is not fixed but follows an inverse demand function $p = p(Q)$. All we need to assume in this case is that the revenue $R(Q) = Qp(Q)$ is a concave function of production Q, which is not unusual. An even weaker assumption, which would also suffice, is that revenue $p(Q(K))Q(K)$ is a concave function of capital K.
- To determine the role of competition by considering an oligopoly rather than atomistic competition. Modelling this situation will lead to a differential game which will not be easy to solve.
- In this paper we studied how input substitution can be used by the firm to respond to environmental regulation. It is important to consider also the other possible reactions to environmental regulation like investing in pollution abatement devices (filters) to clean up, or changing the production process in a more disruptive way than input substitution to reduce emissions.

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Appendix A. Analysis of the adjoint equation at the kinks

First, we note that for $K < \overline{K}$, i.e. case 1, Eq. (44) always reads

$$\dot{\lambda} = (r + a)\lambda - pQ'(K) + v_1 S'(K), \qquad (44a)$$

with

$$S'(K) = \frac{Q'(K)}{G'(S(K))},$$

since $S(K) = G^{-1}(Q(K))$.

In the intermediate interval, i.e., for $\bar{K} < \tilde{K}$ (subcase 2.2), result (39) from Lemma 3 can be used to write Eq. (44) as

$$\dot{\lambda} = (r+a)\lambda - pQ'(K) + BS'(K). \tag{44b}$$

Finally, in the last interval, i.e., for $K > \tilde{K}$ (subcase 3.2), Eq. (44) becomes

$$\dot{\lambda} = (r+a)\lambda. \tag{44c}$$

Now we can specify the intervals occurring in the differential inclusion (45):

The differential inclusion (45) at $K = \bar{K}$ becomes

$$\dot{\lambda} \in \left[(r+a)\lambda - pQ'(\bar{K}) + v_1 S'(\bar{K}),\right.$$
$$\left.(r+a)\lambda - pQ'(\bar{K}) + B \cdot S'(\bar{K})\right] \tag{45a}$$

where we have used Eq. (44a) and Eq. (44b). This interval is non-empty and non-degenerate, because of Lemma 4.

Furthermore, Eq. (45) evaluated at $K = \tilde{K}$ is obtained using Eq. (44b) and Eq. (44c):

$$\dot{\lambda} \in \left[(r+a)\lambda - pQ'(\tilde{K}) + BS'(\tilde{K}), (r+a)\lambda\right]. \tag{45b}$$

This interval is non-empty and non-degenerate because of $pG'(\tilde{A}) > B$ and $S'(K) = Q'(S(K))$.

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